Exercise 10.11

Let $\delta > 0$ be given. Consider the following payment dates

$$\delta, 2\delta, \cdots, (n+1)\delta.$$

Principal amount is equal to 1 and fixed interest rate is K. We consider an interest rate swap where at each payment date $j\delta$, we pay fix interest rate K and receive $L((j-1)\delta, (j-1)\delta)$. Show that the value of this swap at time zero is equal to

$$\delta K \sum_{j=1}^{n+1} B(0, j\delta) - \delta \sum_{j=1}^{n+1} B(0, j\delta) L(0, (j-1)\delta).$$

Proof

We compute the value of each payment at time zero separately and then simply sum them. Note that at time $j\delta$, we pay δK as δ is the tenor, K is the fix interest rate and principal is 1. The value of this payment at time 0 is $\delta K B(0, j\delta)$. On the other hand, at time $j\delta$, $\delta L((j-1)\delta, (j-1)\delta)$ is received. According to **Price of backset LIBOR** theorem from the text, the value of this payment at time 0 is equal to $\delta B(0, j\delta) L(0, (j-1)\delta)$. The result immediately follows.