

**Exercise 10.5 (Correlation between long & short rate in one-factor Vasicek model)**

Show that in one-factor Vasicek model, long rate and short rate are perfectly correlated. Long rate is defined as follows: Fix a relative maturity  $\bar{\tau}$ , long rate at time  $t$  is equal to

$$L(t) = -\frac{1}{\bar{\tau}} \log B(t, t + \bar{\tau})$$

**Proof**

From 4.4.33,

$$R(t) = e^{-bt}R(0) + \frac{a}{b} \left(1 - e^{-bt}\right) + \sigma e^{-bt} \int_0^t e^{bs} dW(s)$$

One-factor Vasicek model has affine-yields. Thus, for deterministic functions  $c(\bar{\tau})$  and  $a(\bar{\tau})$ ,

$$\begin{aligned} L(t) &:= -\frac{1}{\bar{\tau}} \log B(t, t + \bar{\tau}) \\ &= c(\bar{\tau})R(t) + a(\bar{\tau}) \end{aligned}$$

Therefore,

$$\text{Corr}(L(t_2) - L(t_1), R(t_2) - R(t_1)) = \text{Corr}(c(\bar{\tau})[R(t_2) - R(t_1)], R(t_2) - R(t_1)) = 1.$$