Exercise 1.1

Consider a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Show that

- If $A, B \in \mathcal{F}$ and $A \subseteq B$, then $\mathbb{P}(A) \leq \mathbb{P}(B)$
- If $A \subseteq A_n$ and $\lim \mathbb{P}(A_n) = 0$, then $\mathbb{P}(A) = 0$.

Proof

Consider the following set of countable many disjoint sets in $\mathcal{F}$:

$$A, B \backslash A, \emptyset, \emptyset, \emptyset, \cdots.$$

Therefore, $\mathbb{P}(B) = \mathbb{P}(A) + \mathbb{P}(B \backslash A) \geq \mathbb{P}(A)$. To see the second part, note that

$$\mathbb{P}(A) \leq \mathbb{P}(A_n) \quad \forall n.$$

Taking $n \to +\infty$ yields that $\mathbb{P}(A) \leq \lim \mathbb{P}(A_n) = 0$. Thus, $\mathbb{P}(A) = 0$.