Exercise 2.1

Consider a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and a random X on this space. Show that if X is measurable w.r.t the trivial σ-algebra $\mathcal{F}_0 = \{\emptyset, \Omega\}$, then X must be constant.

Proof

Suppose that for $X(\omega_i) = c_i$ for $i = 1, 2$ where $c_1 < c_2$. Let $B = (-\infty, c_2) \in \mathcal{B}$. It must hold that $A = X^{-1}(B) \in \mathcal{F}_0$. However, $\omega_1 \in A$ and $\omega_2 \notin A$. Thus A is not $\emptyset$ neither Ω.